

Relationship between Dimensional Analysis and Similarity

Friction force computation

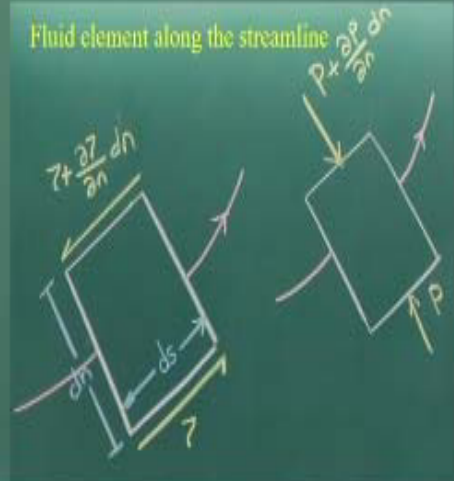
- Force due to the viscosity (friction) is $\frac{\partial \tau}{\partial n} dndsdz$
- The shear force can be expressed as $\mu \left(\frac{\partial^2 |V|}{\partial n^2} \right) dV$
 $dV = dndsdz$

Pressure force computation

- The net pressure force $\left(\frac{\partial p}{\partial n} \right) dV$

Inertia force computation

- For steady flow
 $dma_T = (\rho dV) |V| \frac{\partial |V|}{\partial s}$



Now just to look it, I am not going detail derivations of this part if you take a fluid element along a stimuli like this is the fluid element okay, this is the stream line which is having dx and dn dimensions, you have the shear stress which is changing at this along the n'th directions and you get it what could be the shear stress. Similar way you can find out the pressure values and all.

Force due to the viscosity (friction) is

$$\frac{\partial \tau}{\partial n} dndsdz$$

The shear force can be expressed as

$$\mu \left(\frac{\partial^2 |V|}{\partial n^2} \right) dV$$

Then you can compute the force due to the viscosity that will be the change of shear stress into the volumetric part. That is what if you portrait you get it this part. Rest you substitute the Newton's laws of viscosities and all, then you will get this part.

The net pressure force

$$\left(\frac{\partial p}{\partial n} \right) dV$$

For steady flow, Inertia force computation

$$dma_T = (\rho dV) |V| \frac{\partial |V|}{\partial s}$$

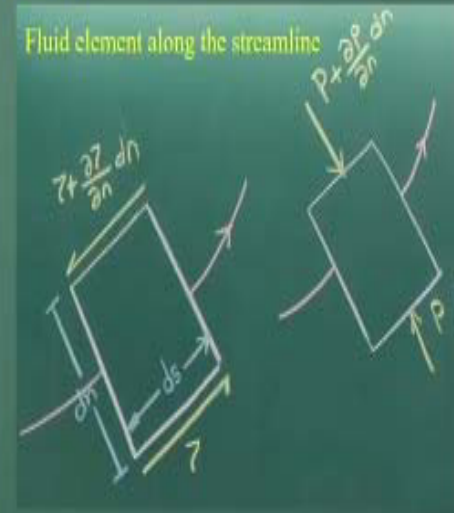
Similar way the net pressure force acting of this you can see it will be this part and inertia force computation which is the, in case of the steady flow, mass into the acceleration or rate of change of the momentum flux that is what the mass and the momentum flux but you compute it, that along the stimuli directions will give us the inertia force components.

(Refer Slide Time: 34:55)

Relationship between Dimensional Analysis and Similarity

- The laws of dynamic similarity the ratio of the forces expressed as

$$\left[\frac{\rho |V| \frac{\partial |V|}{\partial s}}{\mu \frac{\partial^2 |V|}{\partial n^2}} \right]_m = \left[\frac{\rho |V| \frac{\partial |V|}{\partial s}}{\mu \frac{\partial^2 |V|}{\partial n^2}} \right]_p$$

$$\left[\frac{\rho |V| \frac{\partial |V|}{\partial s}}{\frac{\partial p}{\partial n}} \right]_m = \left[\frac{\rho |V| \frac{\partial |V|}{\partial s}}{\frac{\partial p}{\partial n}} \right]_p$$


If you equate it and substitute this values in case of loss of dynamic similarities the ratio between these part, you can see that these equations comes out to be the Reynolds and this equations comes out to be the Euler strength. So basically we are trying to tell it that when you go to the element level we can derive the ratio between inertia force to viscous force which comes out to be Reynolds numbers.

$$\left[\frac{\rho |V| \frac{\partial |V|}{\partial s}}{\mu \frac{\partial^2 |V|}{\partial n^2}} \right]_m = \left[\frac{\rho |V| \frac{\partial |V|}{\partial s}}{\mu \frac{\partial^2 |V|}{\partial n^2}} \right]_p$$

$$\left[\frac{\rho |V| \frac{\partial |V|}{\partial s}}{\frac{\partial p}{\partial n}} \right]_m = \left[\frac{\rho |V| \frac{\partial |V|}{\partial s}}{\frac{\partial p}{\partial n}} \right]_p$$

We can compute it and you have same expressions. Similar way if you are computing the ratio between inertia force to the Euler number which will be reverse of the Euler numbers that what also we will get it and we will get the expressions of the Euler numbers and the Reynolds number. So it is quite easy, you can look at the stream line at the element level as well as the gross characteristic levels we can understand the flow reverse in terms of Reynolds number similarity or the Euler number similarity.

(Refer Slide Time: 36:04)

Example 1

Testing of an automobile vehicle in the Wind Tunnel to find the aerodynamic drag and power required to overcome this drag

Given Data

Model width : 2.44 m
Frontal Area : 7.8 m²
Testing Velocity : 100 km/h
Scale : 16:1
Drag Coefficient : 0.46

To determine the power required for prototype

Parameters

At standard sea level: p=101325 Pa, T=288 K

$$\rho = \frac{p}{RT} = 1.226 \text{ kg/m}^3$$

$$\mu = 1.46 \times 10^{-6} \frac{T^{3/2}}{T + 111} \\ = 1.79 \times 10^{-5} \text{ kg/m.s}$$

Now let us come back to examples like this, let us have a testing of automobiles in a wind tunnel to find the aerodynamic drags, the power required to overcome this drag part. The data is what is given is model width frontal area, testing velocity, the scale, drag coefficient. It is given these data, we need to compute the power required for the prototype level. So since the pressure, the temperatures are given at the standard levels you can find out what will be the density of the air.

Given Data

Model width : 2.44 m

Frontal Area : 7.8 m²

Testing Velocity : 100 km/h

Scale : 16:1

Drag Coefficient : 0.46

To determine the power required for prototype

Parameters

At standard sea level: p=101325 Pa, T=288 K

$$\rho = \frac{p}{RT} = 1.226 \text{ kg/m}^3$$

$$\mu = 1.46 \times 10^{-6} \frac{T^{3/2}}{T + 111} \\ = 1.79 \times 10^{-5} \text{ kg/m.s}$$

You can find out what will be the dynamic viscosity of air which the functions of the temperature, you can compute it this way.

(Refer Slide Time: 36:50)

Step 1	Step 2
<p>Model Width, $w_m = \frac{w_p}{16} = \frac{2.44}{16} = 0.153 \text{ m}$</p> <p>Model Frontal Area, $A_m = \frac{A_p}{16^2} = \frac{7.8}{16^2} = 0.0305 \text{ m}^2$</p> <p>For dynamic similarity:</p> $\left(\frac{\rho VL}{\mu}\right)_m = \left(\frac{\rho VL}{\mu}\right)_p$ $(VL)_m = (VL)_p$ $V_m = \left(\frac{100}{3.6}\right) * \left(\frac{16}{1}\right) = 444.4 \text{ m/s}$	<p>Reynolds Number for model,</p> $(Re)_m = 46.4 * 10^5$ <p>Drag for the prototype:</p> $D_p = C_D \frac{1}{2} \rho V_p^2 A_p$ $= 1697 \text{ N}$ <p>Power to overcome this Drag = $D_p V_p$</p> $= 1697 * \frac{100}{3.6}$ $= 47139 \text{ W}$ $= 47.139 \text{ kW}$

Now you can have the model width and there is a scale ratio is the given for the model width.

We can compute it,

$$\text{Model Width, } w_m = \frac{w_p}{16} = \frac{2.44}{16} = 0.153 \text{ m}$$

$$\text{Model Frontal Area, } A_m = \frac{A_p}{16^2} = \frac{7.8}{16^2} = 0.0305 \text{ m}^2$$

For dynamic similarity:

$$\left(\frac{\rho VL}{\mu}\right)_m = \left(\frac{\rho VL}{\mu}\right)_p$$

$$(VL)_m = (VL)_p$$

$$V_m = \left(\frac{100}{3.6}\right) * \left(\frac{16}{1}\right) = 444.4 \text{ m/s}$$

We use the dynamic similarities means the Reynolds numbers of the models should equal to the Reynolds numbers of the prototypes since the density and the μ .

Reynolds Number for model,

$$(Re)_m = 46.4 * 10^5$$

Drag for the prototype:

$$D_p = C_D \frac{1}{2} \rho V_p^2 A_p$$

$$= 1697 \text{ N}$$

Power to overcome this Drag = $D_p V_p$

$$= 1697 * \frac{100}{3.6}$$

$$= 47139 \text{ W}$$

$$= 47.139 \text{ kW}$$

The same air we are using it so density and the mu are the same so it is come down with this value. So substituting these we can find out what could be the model velocity. Then we can compute it what will be required the Reynolds numbers for the models, because we know the velocity, we can find out what will be Reynolds numbers and what will be the drag force of the prototypes also we can compute it and power requirement of the overcome this drag force also we can compute it using the power is equal to the force into velocity component.

That is what you can use it to compute this part, it is very easy just you have to have a, the many equations is to use is your very simple equations and you have to compute it okay. So like for example, is that we do not know what is the equations of this, you just put the dimensions and check it that whether it is a force component, CD does not have the dimensions, the p square the rho and Fe you just substitute it you will get it the force, the dimensions. Similarly, the power is energy per unit time, you can use this force in to the velocity will be the power.

(Refer Slide Time: 38:56)

Example 2

Problem type: Flow over a sphere [GATE 2017]

Given Data

Type of flow : Laminar flow
 Drag Coefficient : $C_F = F / \rho V^2 D^2$

Density (ρ) = 1000 kg/m³
 Diameter of Sphere (D) = 100 mm
 Velocity (V) = 2 m/s
 CF = 0.5

If water now flows over a another sphere of 200 mm diameter, under DYNAMICALLY SIMILAR conditions, Compute the drag fore on sphere 2

Let us come to the second problem which is flow over a sphere which is GATE 2017 questions which is given laminar flow, drag coefficient equations, the density, diameter of the sphere is given, velocity and the CF value is given. Then water now flows over a sphere of 200 millimetre diameter under the dynamically similar conditions to compute the drag force on the sphere 2.

Given Data

Type of flow : Laminar flow

Drag Coefficient : $C_F = F / \rho V^2 D^2$

Density (ρ) = 1000 kg/m³

Diameter of Sphere (D) = 100 mm

Velocity (V) = 2 m/s

CF = 0.5

(Refer Slide Time: 39:26)

Solution

For dynamically similar conditions

$$Re_1 = Re_2$$

$$\left[\frac{\rho V D}{\mu} \right]_1 = \left[\frac{\rho V D}{\mu} \right]_2$$

$V_1 D_1 = V_2 D_2$ ----- Eqn. 1
 [ρ, μ are not variables]

We know, $C_F = f(Re)$
 and, $C_{F1} = C_{F2}$ ----- Eqn. 2

Force on sphere 2, $F_2 = C_{F2} \rho V_2^2 D_2^2 = C_{F1} \rho V_1^2 D_1^2$
[From Eqn's 1 and 2]

Solving, $F_2 = 20 \text{ N}$

If that is the conditions what will be the this case, there is no free surface, only the pro Reynolds number of model and prototypes we equate it. As it is not given much details about these 2 parameters the density and the mu you can have a basic equations of this ones, just equating them and you can have a this model and the prototypes you have the same CF value. If it is that is the conditions, you can easily compute it force and put the active value.

For dynamically similar conditions

$$Re_1 = Re_2$$

$$\left[\frac{\rho V D}{\mu} \right]_1 = \left[\frac{\rho V D}{\mu} \right]_2$$

$V_1 D_1 = V_2 D_2$ ----- Eqn. 1
 [ρ, μ are not variables]

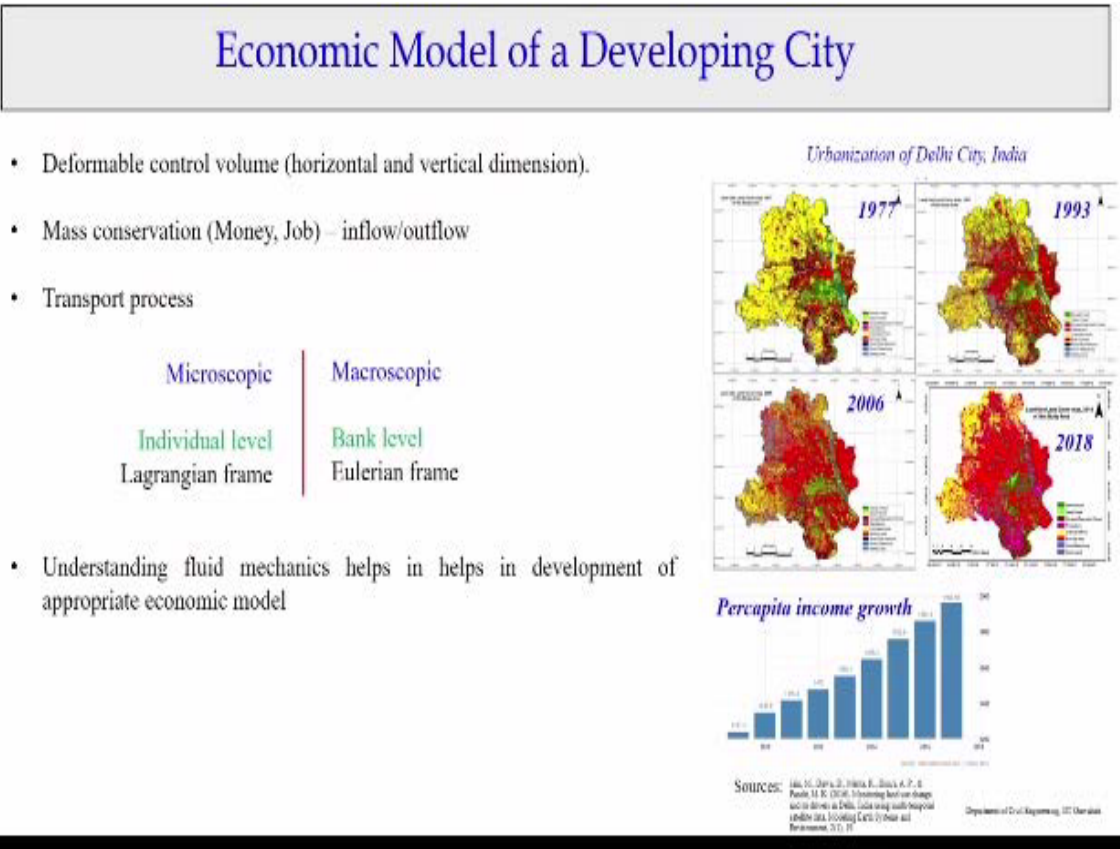
We know, $C_F = f(Re)$
 and, $C_{F1} = C_{F2}$ ----- Eqn. 2

Force on sphere 2, $F_2 = C_{F2} \rho V_2^2 D_2^2 = C_{F1} \rho V_1^2 D_1^2$
[From Eqn's 1 and 2]

Solving, $F_2 = 20 \text{ N}$

So only you have to try to understand it what should be the dynamic similarity will be there, whether the flow, Reynolds number or the flow Froude numbers. Then you will look it at the force levels what is the dynamic similarity conditions like C_{F1} is equal to the C_{F2} which is the functions of the Reynolds numbers. So we can find out the what could be the force acting on this.

(Refer Slide Time: 40:27)



Now let us come back before concluding lectures that most of the time we would feel boring of the fluid mechanics (()) (40:35) lot okay but many of the times the knowledge of the fluid mechanics help us to understand so complex problems like economic model of a developing city. We can use our knowledge of fluid mechanics to develop a economy models for a developing city like Delhi.

If you look at this the red colours are the urban areas, these are real data which is prepared by my students you can see the 1977 what is the red colour area, 1993, 2006, 2018 and this is the real data of economic growth, but we do not know what is going to happen in 2050, 2100, that is what is the economy model students predicted. To do that let us use our fluid mechanics knowledge.

What type of control volume we have, we have deformable control volumes as the control volumes are changing it if you can look it that is control volumes are changing it, both in horizontal and the vertical dimension. Here there are lot of flow is coming, it is mass

conservations we follow it either for money or job, the storage and all, there are lot of transport process happened in economy models okay.

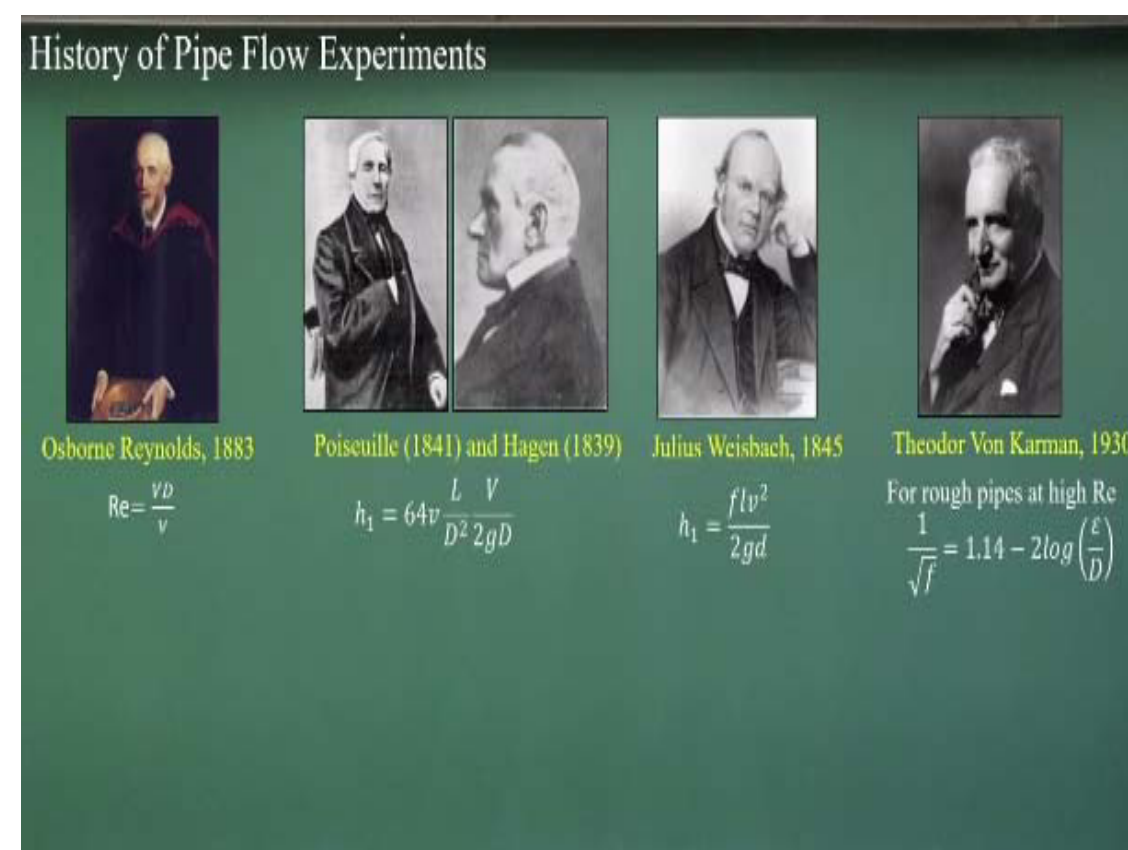
In microscopic level and macroscopic level, the knowledge what we have the fluid mechanics the particles movement, the element movement all these things also used to develop economy model. Similar way we look it, whether a Lagrangian or the Eulerian frame works. Eulerian frame works means we talk about at the bank level, they do not look it individual money, they look t gross, a particular bank is locating what is the total turn overs.

Representing a, so they do not look it the individual part, but individual level a persons is Lagrangian frame work, so that way if you look it if you really understand the fluid mechanics you can solve or you can develop economy models for a developing city like Delhi. So please do have a lot of interest on the fluid mechanics, is not that it is developed just like a the science has developed such a way that with combinations of experiment, theoretical derivations.

The computational techniques, it has come to certain levels that it can also apply many of the fields and most of the fields at the similar concept we follow it and that is what my example to show it. If you are good mathematicians or good fluid mechanic specialist, you can also use the concept of the fluid mechanics to develop a economy models and predict what could be the scenario of the economy or what could be the spread area this red colour area for Delhi city for 2050 or 2100.

With this let us conclude this lecture, but just trying to show it if you look at the history of the pipe flow experiment.

(Refer Slide Time: 43:51)



Which started in 1883 with Reynolds Osbourne, Poiseuille and Hagen okay then Julius Weisbach, 1845 and Von Karman in 1930. So each one they have contributed for pipe flow experiment so uniquely to derive this equations and which we so openly today we are using it and for designing the pipe flows. We will have more discussion of these in the next class. Let me thank you for this lecture today.

Osborne Reynolds, 1883

$$Re = \frac{VD}{\nu}$$

Poiseuille (1841) and Hagen (1839)

$$h_1 = 64\nu \frac{L}{D^2} \frac{V}{2gD}$$

Julius Weisbach, 1845

$$h_1 = \frac{flv^2}{2gd}$$

Theodor Von Karman, 1930

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\epsilon}{D}\right)$$

(Refer Slide Time: 44:32)

Course Teaching Assistants



Vinay Chembolu
Research Scholar
Research Area: Braided River Morphodynamics
Email: vinay140892@gmail.com



Anjaneyulu Akkimi
Research Scholar
Research Area: Urban Hydrology and
Radar Meteorology
Email: anjaneyulu20@gmail.com



Chandan Pradhan
Research Scholar
Research Area: Rivers and Anthropocene
Email: chandanpradhan133@gmail.com